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Robust Cognitive Brain-inspired Computing System: Architectures and Algorithms

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Content

- Introduction
- Our neuromorphic architecture
 - Neuron
 - Processing core
 - 3D Network-on-Chip Integration
- Algorithm and Application
 - Initial mapping solution with Genetic Algorithm
 - Fault-tolerant mapping with Genetic Algorithm
 - Training SNN with ternary weights
 - Application: Multi security-cores control with SNN
- Conclusion

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Brains remain unrivaled computing device



Brain Power: 50mW Weight: 2.2 grams Navigates and learns unknown environments at 35km/h

Can learn to speak words

Can learn to manipulate cups to drink Autonomous Drone



CPU/GPU Power: 18,000mW Weight: 40 grams

Cannot learn anything online

Pretrained to flight between known gates at walking pace

Neuromorphic Computing

- Considered as the third generation of neural networks.
- Mimic the operation of biological brain:
 - Neuron communicates via action potentials (spikes),
 - Neuron integrates the action potentials into its membrane potential,
 - Once the membrane potential crosses the threshold, neuron fires (issues spikes)
 - Action potentials travel from upstream neuron to downstream neuron via axon-dendritesynapse
 - The connections between neurons have different strengths (weights)



Neuromorphic Computing: Benefits

- Near Data/Memory Computing
- Sparse connections (spatial and temporal)
- On-chip learning without weight movement and data storage
- Power and area efficient:
 - No floating-point unit
 - No multiplication (discuss later)
 - No off-chip DRAM

Notable existing works

- Neurogrid by Stanford University:
 - Mixed signal neuron design: capacitor as neuron to integrate the incoming spikes (current).
 - Firing by voltage comparator
- BrainscaleS (European Union project):
 - Also mixed signal neuron design with leaky function (leak current)
 - Hierarchical routing structure
- SpikNNaker by University of Manchester:
 - One million ARM968 cores system
 - Each core simulation 1000 neurons
 - Supercomputer-like structure
- TrueNorth by IBM:
 - 2D-Network-on-Chip based system
 - Integrate and Fire neuron (digital)
 - 256 neuron per node, 256 input, 64k synapses/node
 - 1-bit weight, offline learning
- Loihi by Intel:
 - 2D-Network-on-Chip based system
 - Programmable neuron model (digital)
 - Variable bit width for weight

A comparison between human brain and largescale system

Platform: Human brain		Neurogrid	BrainScaleS	TrueNorth	SpiNNaker	
Technology:	Biology	Analogue, sub- threshold	Analogue, over threshold	Digital, fixed	Digital, programmable	
Microchip:		Neurocore	HiCANN		18 ARM cores	
Feature size:	$10 \mu m^{a}$	180 nm	180 nm	28 nm	130 nm	
# transistors:		23 M	15 M	5.4 B	100 M	
die size:		$1.7 \mathrm{cm}^2$	0.5 cm^2	4.3 cm^2	1 cm^2	
# neurons:		65 k	512	1 M	16 k	
# synapses:		~100 M	100 k	256 M	16 M	
power:		150 mW	1.3 W	72 mW	1 W	
Board/unit:		PCB	20 cm wafer	PCB	PCB	
# chips:		16	352	16	48	
# neurons:		1 M	200 k	16 M	768 k	
# synapses:		4 B	40 M	4B	768 M	
power:		3 W	500 W	1 W	80 W	
Reference system:	1.4 kg		20 wafers in 7 × 19" racks		600 PCBs in 6 × 19" racks	
# neurons:	100 B		4 M		460 M	
$\#$ synapses: 10^{15}			1 B		460 B	
power:	20 W		10 kW		50 kW	
Energy/connection:	10 fJ	100 pJ	100 pJ	25 pJ	10 nJ	
Speed versus biology:	$1 \times$	1×	10 000×	1×	1×	
Interconnect:	3D direct signalling	Tree-multicast	Hierarchical	2D mesh-unicast	2D mesh-multicast	
Neuron model:	Diverse, fixed	Adaptive quad- ratic IF	Adaptive exponen- tial IF	LIF	Programmable ^b	
Synapse model:	Diverse	Shared dendrite	4-bit digital	Binary, 4 modulators	Programmable ^c	
Run-time plasticity:	Yes!	No	STDP	No	Programmable ^d	

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Our neuromorphic platform

- Hardware:
 - A completed neuromorphic system
 - Neuron: Leaky-Integrate-and-Fire (LIF) model in fully parallel mode
 - Synapses: parallel SRAMs
 - Axon: 3D-Network-on-Chip communication
 - Learning: off-chip (ANN-SNN conversion), on-chip (bio-plausible STDP)
- Software:
 - Mapping neuromorphic to hardware using Genetic Algorithm
 - Tolerating faults in neurons using Genetic Algorithm
 - Ternary weight training for MLP and CNN
 - Application: SNN controller for multi AES-cores system

Our platform



Architecture of the neuromorphic system



Leaky Integrated and Fire Neuron

• Computation:

$$V_j(t) = V_j(t-1) + \sum_i w_{i,j} x_i(t-1) - \lambda$$

- $V_j(t)$: the membrane potential of neuron *j* at time step *t*
- *w_{i,j}*: the synapse weight between neuron *i* and neuron j
- $x_i(t-1)$: the output of the presynaptic neuron i
- λ : leaky value.
- The output of a neuron is:

$$x_{j}(t) = \begin{cases} 1 \ if \ V_{j}(t) > V_{thres} \\ 0, \quad otherwise. \end{cases}$$



On-chip interconnect



- Use our 3D Network-on-Chip platform: stacking 3D-IC, 3D-Mesh topology, interlayer using through-silicon-vias (TSV)
- Two types of packets:
 - Spike packet: include the AER for PE (SNPC) and neuron.
 - Spike packet also includes mask for sparsity
 - Memory access: to read and write memory.

Hardware Implementation

- Hardware Architecture:
 - Designed in Verilog HDL
 - Designed with commercial CAD tools with NANGATE45nm
 - TSV integration using FreePDK3D45
 - OpenRAM for SRAM generation



Layout of a 2x2 NoC-based SNN layer with migration support. tile's size is $790\mu m \times 1580\mu m$.

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[1] Initial mapping

- When deploying, initial mapping (placing neurons and connecting them) is one of the major problems
 - NoC mapping is NP-completed problem
- We provide a Genetic-Algorithm based solution:
 - Optimize for communication cost.
 - Could be easily extended.



[1] Initial mapping for SNN using Genetic Algorithm

Algorithm 1: Proposed Genetic Algorithm for Neurons Mapping

1 2 3	// initialize phaseS1: load the system configuration;S2: randomize the K mapping solutions;// evolve phasefor (generation g_i in 1 to G) do	$T_{cost} = \sum_{i=0,j=0}^{W} d_{ij} \times c_{ij}$	
4 5 6 7 8	S3: remove the wrong mapping solutions; S4: calculate cost function (communication cost) for d_{ij} : S5: select the <i>B</i> best out of <i>K</i> solutions based on t S6: mutate the <i>B</i> best solutions to have new <i>K</i> sol S6: crossover the new <i>K</i> solutions to have new pop S7: check if it satisfies the communication cost or S6: C_{ij} : C	distance between neuron i and ron j connection between neuron i and ron j	;
9 10	// finalize phase S7: calculate cost function for each solution of the population S8: select the $B = 1$ best out of K solutions based on the co	on; ost function:	

Abderazek Ben Abdallah, **Khanh N. Dang**, *"Towards Robust Cognitive 3D Brain-inspired Cross-paradigm System"*, **Frontiers in Neuroscience**, Frontiers, Volume 15, pp. 795, 2021. [DOI: 10.3389/fnins.2021.690208]

Crossover and Mutation



Crossover





Mutation

Initial mapping with GA



We compare to the linear mapping method from SpiNNaker and implement it for the 3D topology to get Linear X, Linear XY, and Linear XYZ

Genetic Algorithm Result for initial mapping. (A) 4 × 4 × 4 NoC-based, 256 neurons/node. (B) 6 × 6 × 6 NoC-based, 256 neurons/node. (C) 8 × 8 × 8 NoC-based, 256 neurons/node.(D) 10 × 10 × 10 × 10 NoC-based, 256 neurons/node.

[2] Fault-tolerant mapping

• After deploying, there is a probability of neuron failure (faults in memory, neuron or controller).



Khanh N. Dang, Nguyen Anh Vu Doan, Abderazek Ben Abdallah *"MigSpike: A Migration Based Algorithm and Architecture for Scalable Robust Neuromorphic Systems"*, **IEEE Transactions on Emerging Topics in Computing (TETC)**, <u>in-press</u> [DOI: 10.1109/TETC.2021.3136028]

[2] Fault-tolerant mapping with GA

- Similar to initial mapping, GA is also used to solve the fault-tolerant mapping.
 - Two cost function: (1) migration cost: movement of neurons during the correction phase and (2) communication cost: travelling distance of spikes
- Crossover is performed by mixing two parents with adjustments to make sure the right amount of neuron being mapped.
- Mutation by reducing the migration cost

Fault-tolerant Mapping Result



 Output mapping for migrated neurons with random fault patterns in 3D-NoCs. The system has 256 neurons per node; 20% of neurons are spare with 1 redundant node without any allocated neuron at 0% fault rate.

[3] Training for SNN

- Training for Neuromorphic Systems:
 - Spike-timing-dependent plasticity (STDP): a bio-plausible learning method
 - Support online learning with hardware STDP block.
 - Limitation: single-layer (local) training and low accuracy for classification tasks
 - ANN-SNN conversion:
 - Train with ANN first.
 - Convert to SNN by shifting the computation domain.
 - Can be trained with deep neural networks and maintain high accuracy and low power features



Spike-timing-dependent plasticity (STDP) Learning Module

STDP for MNIST



Network: 784:N with lateral inhibitory connections in the second layer.

N=100 \rightarrow accuracy = 71.32%, N=400 \rightarrow accuracy = 84.05%

[3] Training SNN with ternary weight in ANN-SNN conversion

- Neural networks system usually trained with float values (weight, bias) that requires 32 bit to represent.
- Quantization process can covert these values into fixed point format (8/16 bit) to reduce the storage and computation complexity.
- We can quantize it into ternary value (-α, 0, α) (α is fixed) that require 2-bit to represent. → This ternary process has been done with DNN. We proposed a training method for SNN.

Duy-Anh Nguyen, Xuan-Tu Tran, **Khanh N. Dang**, and Francesca Iacopi, *"A Low-Power, High-Accuracy with Fully On-Chip Ternary Weight Hardware Architecture for Deep Spiking Neural Networks"*, **Microprocessors and Microsystems**, 2022 (inpress).

Training process with ternary weights



Ternary process

• Ternary weight:

$$w_{L}^{tern} = \begin{cases} \alpha \times Sign(w_{L}^{fp}) if |w_{L}^{fp}| \geq \Delta_{th} \\ 0 & otherwise. \end{cases}$$

• α is the Mean of absolute values:

$$\alpha = E(|w_L^{fp}|) \forall \{|w_L^{fp}| \ge \Delta_{th}\}$$

• Threshold:

$$\Delta_{th} = \beta \times \max(|w_L^{fp}|)$$

Note: $\beta = 0.01$ in our experience.

Fully Connected Network Models

Network name	Network type	Network configuration	Weight Precision
FC256	Fully Connected	784-256-10	32b
FC256_TW	Fully Connected	784-256-10	2b
FC256_256	Fully Connected	784-256-256-10	32b
FC256_256_TW	Fully Connected	784-256-256-10	2b
FC1024	Fully Connected	784-1024-10	32b
FC1024_TW	Fully Connected	784-1024-10	2b
FC1024_1024	Fully Connected	784-1024-1024-10	32b
FC1024_1024_TW	Fully Connected	784-1024-1024-10	2b







Convolutional Neural Network Models

Architecture	DNN	DNN-SNN (converted, 32b FP)	TW-SNN (2b)
VGG5	88.61%	87.21% (T=250)	85.10% (T=250)
VGG9	90.26%	89.58% (T=250)	87.58% (T=250)
VGG13	91.38%	90.00% (T=250)	89.47% (T=250)
VGG16	91.63%	90.34% (T=250)	89.71% (T=250)

CIFAR-10 dataset

Model	Training method	Architecture	Accuracy	Timesteps
Hunsberger et al. (2015)[56]	DNN-SNN Conversion	2 Conv, 2 Linear	82.95%	6000
Cao et al. (2015)[28]	DNN-SNN Conversion	3 Conv, 2 Linear	77.43%	400
Sengupta et al. (2019)[23]	DNN-SNN Conversion	VGG16	91.55%	2500
Lee et al. (2020)[57]	Spiking Backpropagation	VGG9	90.45%	100
Park et al. (2019)[58]	DNN-SNN Conversion	VGG16	91.41%	793
This work	TW-SNN	VGG16	89.71%	250

Comparison using CIFAR-10 dataset



[4] Application: Multi security-cores control with SNN

- The baseline slide system consists of 10 AES cores which allow extremely high bandwidth and adaption.
- To reduce the power consumption, we apply clock-gating and power-gating to each core.
- However, the decision process to turn on/turn off a core must be considered:
 - Turn on too many core \rightarrow waste power consumption.
 - Turn off too many core → cannot satisfy the demanded throughput.
- We design our own dataset for the throughput adaptation and train SNN to control:
 - SNN has low complexity \rightarrow extremely small controller.
 - Control the system (change the number of cores) in every T cycles
 - Can be turn off to save power.

Dong-Khoi Pham et al. "A Low-power multicore AES system with neuromorphic controller", (submitted)

The software/hardware flows



Example of training data (9 out of 27)



Input data speed

Enable Cores

Input data speed

Enable Cores

when

- 10

- 5

SNN training and architecture



SNN training: (a)The ANN-to-SNN conversion flow and (b)The quantization flow (8-bit)





SNN hardware architecture:

SNN control results







aes fc

[12.5% dataset]

98.326%

100

MAX:100.000% T=15

Area Cost Full System		
Modules	Area (μm^2)	Percent
System	991,987.4234	100
MAESx10	897903.757	
MAESx10/DEMUX1	35733.641	
MAESx10/MUX	35811.313	
SNN-CONTROLLER	22,537.3824	2.27

Error cases are still acceptable with ± 1 from the idea number of

core

8-bit fixed point accuracy: 95%

Area cost

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Conclusion

- We proposed a hardware architecture for neuromorphic computing:
 - Support LIF neuron
 - Clusterize neurons into node
 - Connected via 3D-Network-on-Chip
- We support offline and online training for the neuromorphic system.
- Augmented algorithms:
 - Initial mapping with Genetic Algorithm
 - Fault-tolerant mapping Genetic Algorithm.
- Future works:
 - Advanced memory technology: RRAM, STT-RAM, PWM.
 - Advanced 3D-IC: monolithic 3D.

Reference

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Thank you for your attention!

Backup Slides

Just in case ;)

Notable existing works



Qualcomm Zeroth (2013)



IBM TrueNorth (2014)



Intel Loihi (2017)



BrainChip (2019)



Learning for neuromorphic system

- Offline learning with backpropagation trained ANN-to-SNN conversion:
 - Feed-forward network is trained with back-propagation.
 - Weight normalization and conversion is performed.
 - Weights and parameters are exported to fixed bit format.
 - Deploy by downloading the weights and parameters.
- Online learning with STDP (Spike-Timing-Dependent Plasticity):
 - Follow the Hebbian learning rule and divide into Long-Term-Depression (LTD) and Long-Term-Potentiation (LTP)
 - LTP: if the spike on the synapse occurs before firing (cause the firing), the strength of synapse increase.
 - LTD: if the spike on the synapse occurs after firing, the strength of synapse increase.
 - Supported by hardware STDP block.

Near Data Processing Approach



- The "integration" and "leaky" of LIF neuron is performed by an adder
- The most challenging part is the weighted input (product of input spikes and weight)
- Conventional ANN uses multipliers as a part of its MAC (multiplication and accumulation)

Near Data Processing Approach (cnt.)

- In our design, multiplication is converted to memory accessing
- $x_i(t-1)$, as binary input, is converted to memory access.
- If $x_i(t 1) = 0$, no access (return 0)
- If $x_i(t-1) = 1$, By accessing the address of $w_{i,j}$, we can extract the value of



 $w_{i,j} x_i (t-1)$

STDP



STDP: update rule



- The adjustment window is 16 timesteps.
- If neuron j fires, STDP update initiates.
- Presynaptic neuron i fires within 1 to 8 timesteps before the firing time of neuron j $\rightarrow w_{i,j}$ increases
- Presynaptic neuron i fires within 0 to 7 timesteps after the firing time of neuron j $\rightarrow w_{i,j}$ decrease

ANN-to-SNN conversion for MNIST



STDP for MNIST

Α



Network: 784:N with lateral inhibitory connections in the second layer.

N=100 \rightarrow accuracy = 71.32%, N=400 \rightarrow accuracy = 84.05%

Fault-tolerant mapping: Execution time

	Size	Remap	1-hop GS	N-hop GS	\mathbf{GA}^2	MFMC	Load Balancing [41]	Kernighan-Lin [41]
x	[4,4]	$20.1 \ \mu s$	90.3 μs	$113 \ \mu s$	$5.916 \ s$	$351.4 \ \mu s$	-	-
	[6,6]	$38.2 \ \mu s$	$378.2 \ \mu s$	$421.3 \ \mu s$	$53.826 \ s$	$672.9 \ \mu s$	-	-
Q	[8,8]	74.3 μs	$0.640 \ ms$	$1.155 \ ms$	$140.392 \ s$	$1.202 \ ms$	-	-
ž	[10, 10]	$216.5 \ \mu s$	$1.526 \ ms$	$2.076 \ ms$	$327.070 \ s$	$2.640 \ ms$	-	-
D	[12,12]	$369 \ \mu s$	$3.413 \ ms$	$3.499 \ ms$	$640.914 \ s$	$5.032 \ ms$	-	-
5	[14,14]	$608 \ \mu s$	$5.438 \ ms$	$6.150 \ ms$	$1220.911 \ s$	$7.428 \ ms$		-
	[16, 16]	932.5 μs	$7.698 \ ms$	$8.713 \ ms$	$1932.054 \ s$	$11.097 \ ms$	-	-
	[4,4,4]	$39.2 \ \mu s$	780.100 μs	942.8 μs	$141.509 \ s$	$1.276 \ ms$	-	
00	[6,6,6]	116.3 μs	$7.349 \ ms$	$8.387 \ ms$	$1498.355 \ s$	8.771ms	-	-
Q	[8,8,8]	$394.5 \ \mu s$	$13.487 \ ms$	$14.076 \ ms$	3.178 h	24.604 ms	-	-
ž	[10, 10, 10]	$1.200 \ ms$	32.151 ms	$44.706 \ ms$	$15.237 \ h$	$63.545 \ ms$	-	-
Q	[12, 12, 12]	$3.365 \ ms$	$101.105 \ ms$	$106.223 \ ms$	N/A ³	$116.658 \ ms$		-
3	[14, 14, 14]	$8.333 \ ms$	$184.655 \ ms$	204.551 ms	N/A^{-3}	$208.800 \ ms$	-	-
	[16, 16, 16]	$19.938 \ ms$	$236.766 \ ms$	$291.288 \ ms$	N/A ³	$442.986 \ ms$	(-)	-
Any	W=1000, E=128	$14.3 \ \mu s$	44.6 μs	$65.400 \ \mu s$	3.856 s	$310.1 \ \mu s$	4.570 s	$2030.1 \ s$

⁰ The execution only takes into account the computation time for finding the new mapping. Other calculations such as setting up or configuration generation are not counted.

^I System configuration: E=256, $W = 0.8^*X$, $k = 0.2^*X$.

² Genetic Algorithm configuration: 100 parents, 20 bests, 40 crossover and 40 mutations per generation, 200 generations.

³ GA in 3D-NoC with the size from [12,12,12] is infeasible to perform due to extremely long execution time and large allocated memory.

⁴ Linear mapping: the neurons with lower indexes are mapped to the nodes with lower indexes.

Initial mapping with GA: 2D vs 3D



 Comparison between 3D and 2D mapping. (A) 64 nodes (4 × 4 × 4 and 8 × 8) NoC-based, 256 neurons/node. (B) 128 nodes (4 × 4 × 8 and 8 × 16) NoC-based, 256 neurons/node. (C) 256 nodes (4 × 8 × 8 and 16 × 16) NoC-based, 256 neurons/node.(D) 512 nodes (8 × 8 × 8 and 16 × 32) NoC-based, 256 neurons/node.

Mapping solution: Max-Flow Min-Cut



- For all nodes, create a flow of migration with the capacity is the maximum migrating neurons they can perform.
- All neurons can travel d_{max} hops \rightarrow connect each node to all nodes in d_{max} hops.
- Flow from virtual source to each node: number of faulty neurons
- Flow from each node to virtual sink: number of spares
- Solve the max-flow problem using Ford-Fulkerson

Example: input

	 act 	active neurons • faulty neurons • inactive neurons • unmapped neurons 34 flow of mapping					fmapping		
(0,0,0)	(0,0,1)	(0,0,2)	(1,0,0)	(1,0,1)	(1,0,2)	(2,0,0)	(2,0,1)	(2,0,2)	
×252 ×0 ×4 (0,1,0)	×210 ×42 (0,1,1)	×252 ×0 ×4 (0,1,2)	×236 ×16 ×4 (1,1,0)	×252 ×0 (1,1,1) ×4	×252 ×0 (1,1,2) ×4	× 197 × 55 × 4 (2,1,0)	×252 ×0 (2,1,1) ×4	×252 ×0 (2,1,2) ×4	
×251 ×5 ×4 (0,2,0)	×252 ×0 (0,2,1) ^{×4}	×252 ×0 (0,2,2) ×4	×252 ×0 (1,2,0) ×4	×252 ×0 (1,2,1) ^{×4}	×252 ×0 (1,2,2) ×4	×252 ×0 (2,2,0) ×4	×252 ×0 (2,2,1) ^{×4}	×252 ×0 (2,2,2) ×4	
×252 ×0 ×4 (0,3,0)	×252 ×0 (0,3,1) ×4	×252 ×0 (0,3,2) ×4	×252 ×0 (1,3,0)	×252 ×0 (1,3,1) ×4	×252 ×0 (1,3,2) ×4	×248 ×4 ×4 (23,0)	×236 ×16 (2,3,1)×4	×252 ×0 (2,3,2) ×4	
● ×252 ● ×0 ● ×4	● ×252 ● ×0 ● ×4		● ×252 ● ×0 ● ×4	● ×252 ● ×0 ● ×4	◆ ×252 ◆ ×0 ◆ ×4	● ×252 ● ×0 ● ×4	● ×252 ● ×0 ● ×4	● ×240 ● ×0 ● ×16	
	Layer 0			Layer 1			Layer 2		

Example: output



Mutation for GA in fault-tolerant mapping

