Automata and Languages

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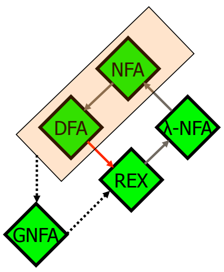
Today's Topics

- **DFA to Regular Expression**
- **GFNA**
- **DFA→GNFA**
- **GNFA → RE**
- **DFA → RE**
- **Examples**

DFA → RE



GNFA

Definition

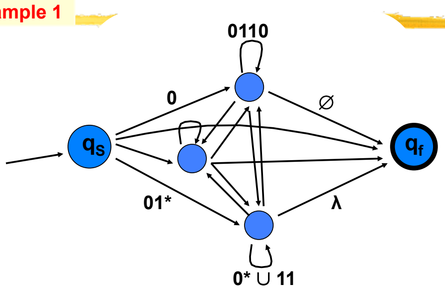
A ***generalized nondeterministic finite automaton*** (**GNFA**) is a graph whose edges are labeled by regular expressions, with a unique start state with in-degree 0, and a unique final state with out-degree 0.

A string *u* is said to ***label*** a path in a GNFA, if it is an element of the language generated by the regular expression which is gotten by concatenating all labels of edges traversed in the path.

The ***language accepted*** by a GNFA consists of all the accepted strings of the GNFA.

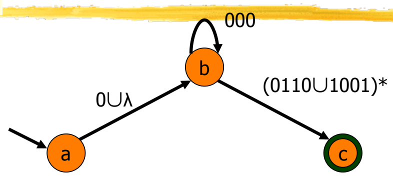
GNFA

Example 1



GNFA

Example 2

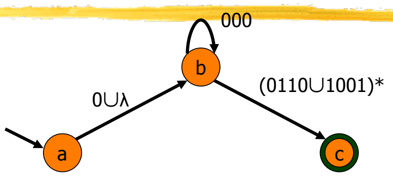


This is a GNFA because edges are labeled by REX’s, start state has no in-edges, and the *unique final* state has no out-edges.

Q: Is 000000100101100110 accepted?

GNFA

Example 2



A: 000000100101100110 is accepted. The path given by $a \xrightarrow[\lambda]{} b \xrightarrow[000]{} b \xrightarrow[000]{} b \xrightarrow[100101100110]{} c$ proves this fact.

(The last label results from 100101100110 ∈ (0110∪1001)*)

GNFA

Definition

A Generalized nondeterministic finite automaton (GNFA) is defined by $M=(Q, \Sigma, \delta, q_{start}, q_{final})$ with

- Q finite set of states
- Σ the input alphabet
- q_{start} the start state with 0 in-degree
- q_{final} the accept state with 0 out-degree

- $\delta:(Q\setminus\{q_{final}\})\times(Q\setminus\{q_{start}\}) \to R$ the transition function

- (R is the set of regular expressions over Σ)

DFA→GNFA

Theorem

For every DFA M, there exist an equivalent GNFA M'

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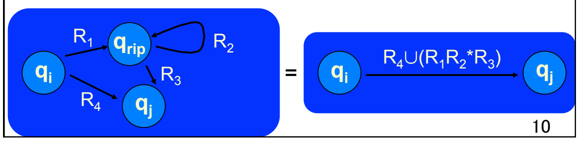
DFA→GNFA

Remove Internal state of GNFA

If the GNFA M has more than 2 states, 'rip' internal q_{rip} to get equivalent GNFA M' by:
- Removing state q_{rip}: $Q' = Q \setminus \{q_{rip}\}$
- Changing the transition function δ by

$$\delta'(q_i,q_j) = \delta(q_i,q_j) \cup (\delta(q_i,q_{rip})(\delta(q_{rip},q_{rip}))^*\delta(q_{rip},q_j))$$

for every $q_i \in Q' \setminus \{q_{finalt}\}$ and $q_j \in Q' \setminus \{q_{start}\}$



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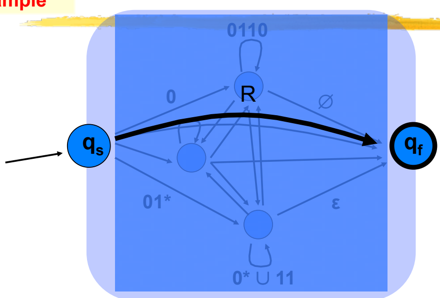
GNFA→RE

Example



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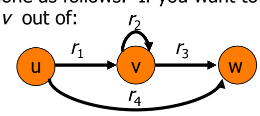
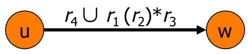
GNFA→RE

Ripping out

Ripping out is done as follows. If you want to rip the middle state v out of:



then you'll need to recreate all the lost possibilities from u to w. I.e., to the current REX label r_4 of the edge (u,w) you should add the concatenation of the (u,v) label r_1 followed by the (v,v)-loop label r_2 repeated arbitrarily, followed by the (v,w) label r_3. The new (u,w) substitute would therefore be:



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DFA→RE

Theorem

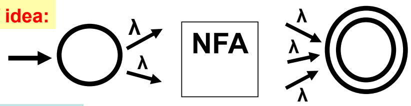
For every (NFA) DFA *M*, there exist an equivalent RE *R,* with L(M)=L(R)

Proof idea:

1. Transform an NFA into an equivalent GNFA

2. Transform the GNFA into a regular expression by removing states and re-labeling the arrows with regular expressions

Proof idea:



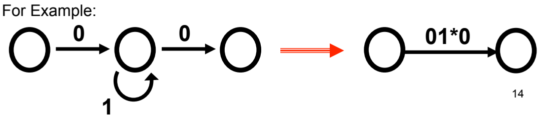
1. NFA→GNFA

Add unique and distinct start state with 0 in-degree and a final state with 0 out-degree, then connect them to the NFA with λ

2. GNFA→RE

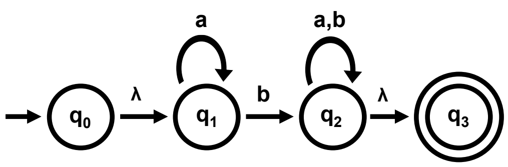
While machine has more than 2 states:
Pick an internal state, rip it out and re-label the arrows with regular expressions to account for the missing state

For Example:



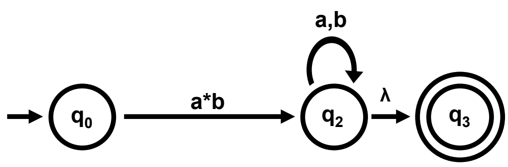
DFA→RE

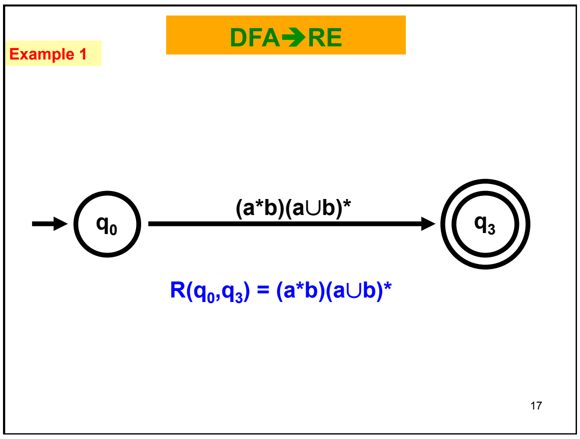
Example 1

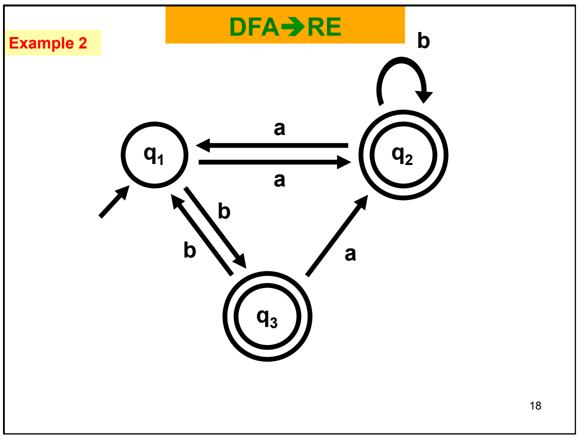


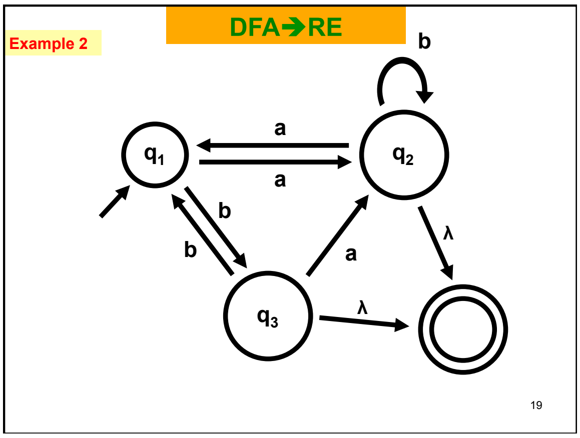
DFA→RE

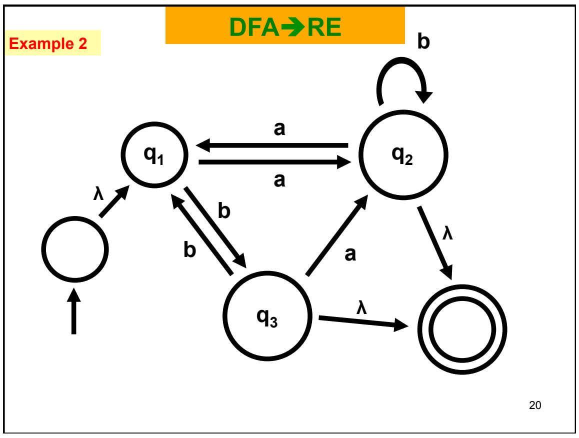
Example 1

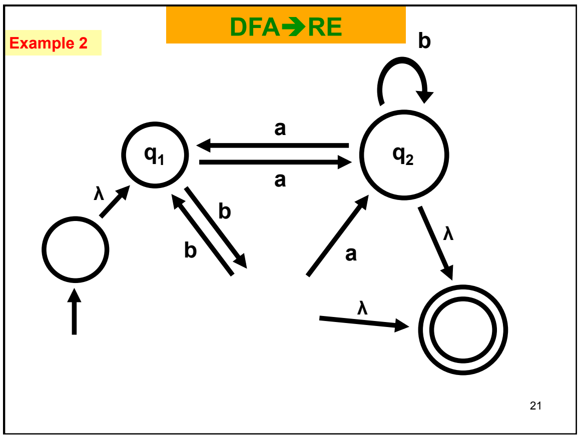



DFA➔RE
Example 1
(a*b)(a∪b)*
q_0
q_3
R(q_0,q_3) = (a*b)(a∪b)*
17

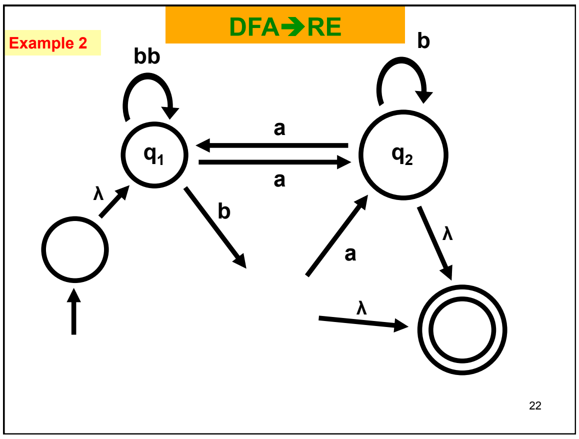

DFA➔RE
Example 2
b
a
a
q_1
q_2
b
b
a
q_3
18


DFA➔RE
Example 2
b
a
a
q_1
q_2
b
b
a
λ
q_3
λ
19

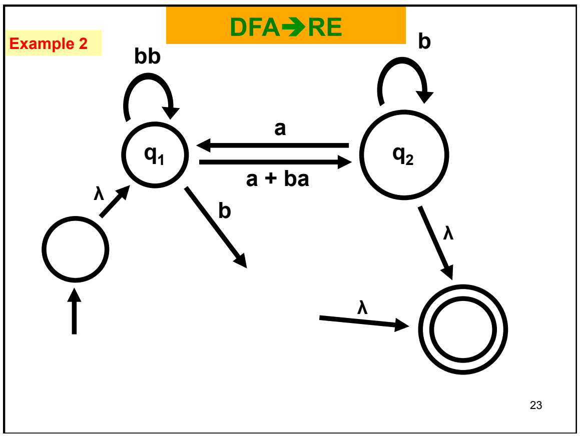

DFA➔RE
Example 2
b
a
a
q_1
q_2
λ
b
b
a
λ
q_3
λ
20



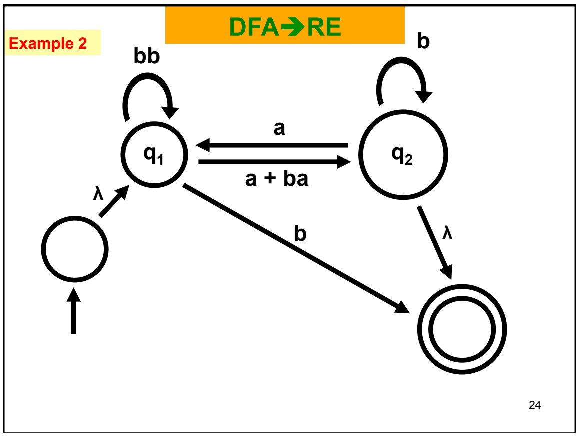
Example 2
DFA→RE
b
a
a
b
b
a
λ
λ
λ
21



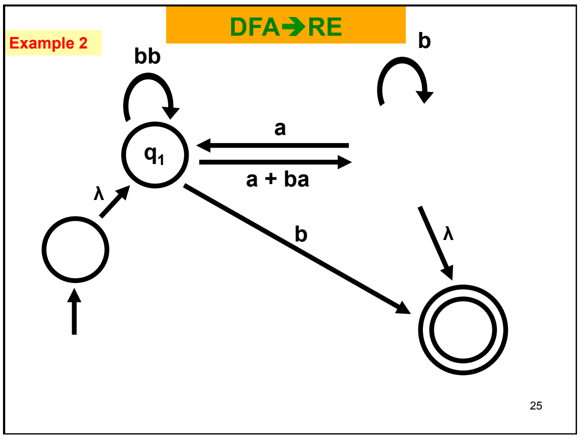
Example 2
DFA→RE
bb
b
a
a
b
a
λ
λ
λ
22

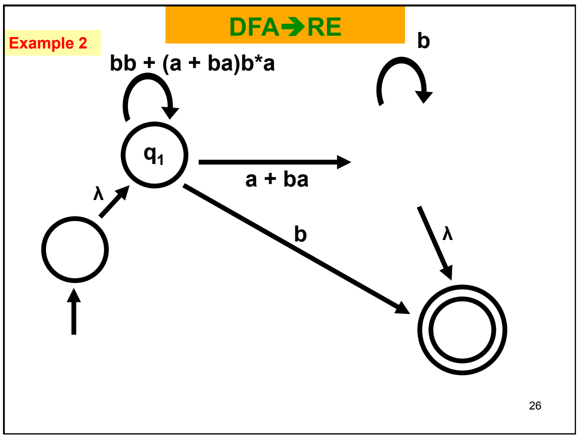


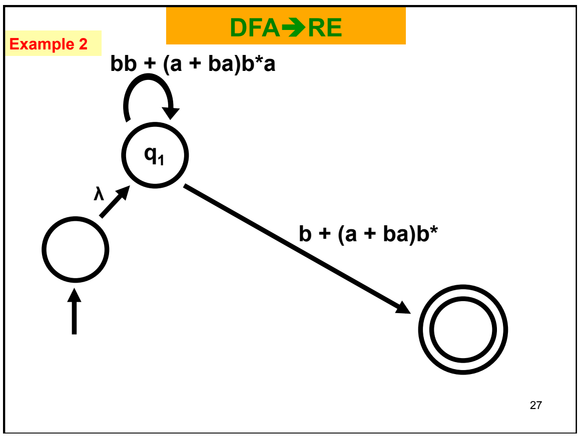
Example 2
DFA→RE
bb
b
a
a + ba
b
λ
λ
λ
23

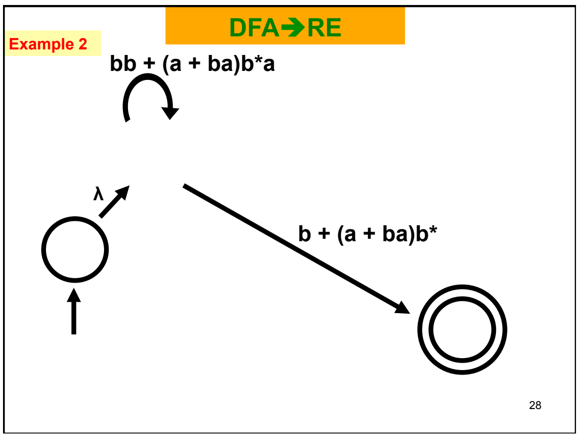


Example 2
DFA→RE
bb
b
a
a + ba
b
λ
λ
24


Example 2
DFA→RE
bb
b
q1
a
a + ba
λ
b
λ
25

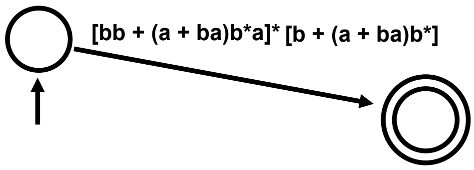

Example 2
DFA→RE
bb + (a + ba)b*a
b
q1
a + ba
λ
b
λ
26


Example 2
DFA→RE
bb + (a + ba)b*a
q1
λ
b + (a + ba)b*
27


Example 2
DFA→RE
bb + (a + ba)b*a
λ
b + (a + ba)b*
28

Example 2

DFA➔RE



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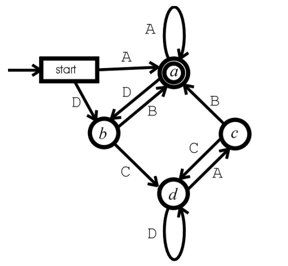
Example 3

DFA➔RE

Start with:

This is not a GNFA because final state, though unique, has nonzero out-degree.

First we need to convert this into a GNFA.



Start state has zero in-degree, so is okay.

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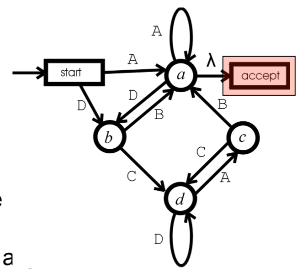
Example 3

DFA➔RE

Just added an accept state by connecting via λ from the old accept state.



Since the labels are single letters, they are automatically regular expressions so this is a GNFA And we are ready to start ripping out interior states.

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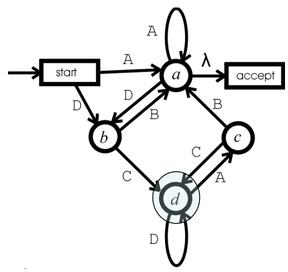
Example 3

DFA➔RE

Now, let' s rip out *d*.

Q1: What will be the label of (*b*,*c*)?

Q2: What will be the label of (*c*,*c*)?



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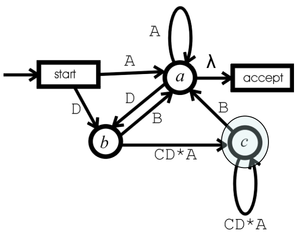
Example 3

DFA→RE

A1: (*b*,*c*): **CD*A**
A2: (*c*,*c*): **CD*A**



Next, rip out *c.*

Q: What will be the label of (*b*,*a*)?

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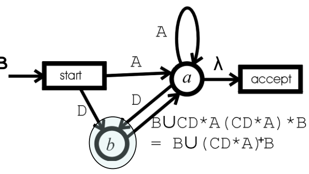
Example 3

DFA→RE

A:
B∪(CD*A)(CD*A)*B
which simplifies to
B∪(CD*A)$^{+}$B



Next, rip out *b.*

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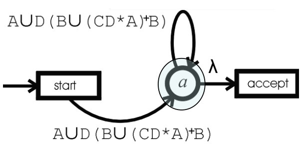
Example 3

DFA→RE



Finally, rip out *a* :

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Example 3

DFA→RE

The resulting REX
Is the unique
label
(A∪D(B∪(CD*A)$^{+}$B))$^{+}$

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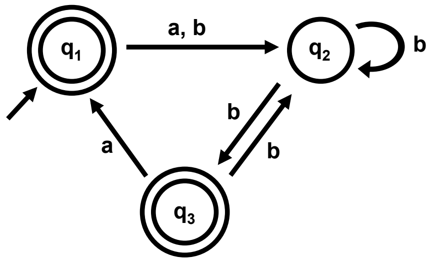
Example 4

DFA→RE

Convert the following NFA into a regular expression?



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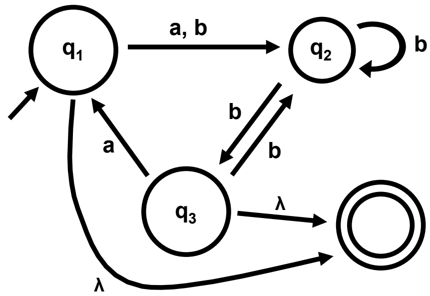
Example 4

DFA→RE

Convert the following NFA into a regular expression?



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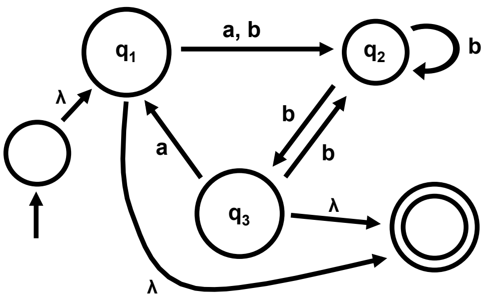
Example 4

DFA→RE

Convert the following NFA into a regular expression?



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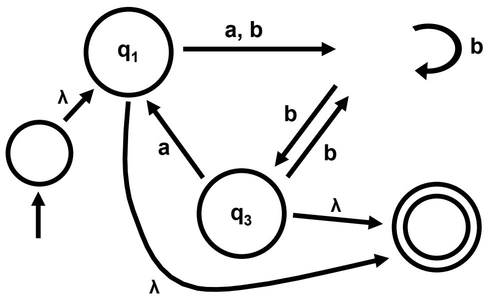
Example 4

DFA→RE

Convert the following NFA into a regular expression?



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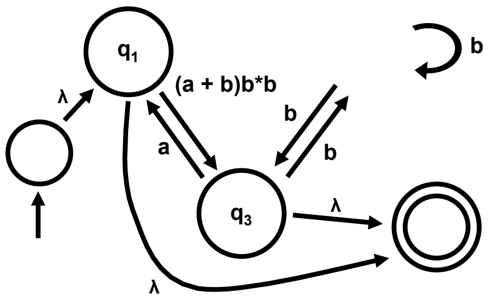
Example 4

DFA→RE

Convert the following NFA into a regular expression?



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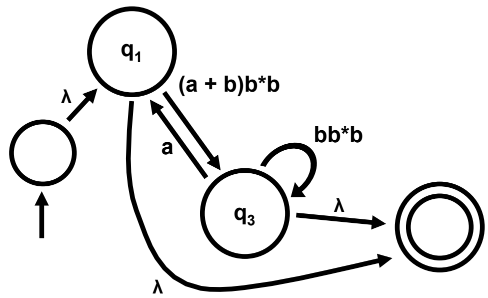
Example 4

DFA→RE

Convert the following NFA into a regular expression?



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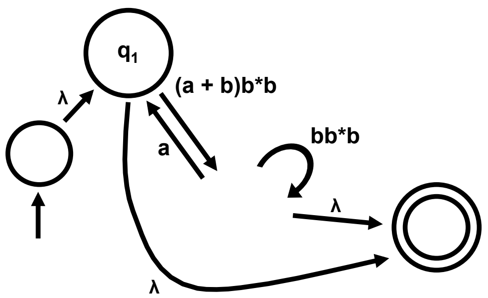
Example 4

DFA→RE

Convert the following NFA into a regular expression?



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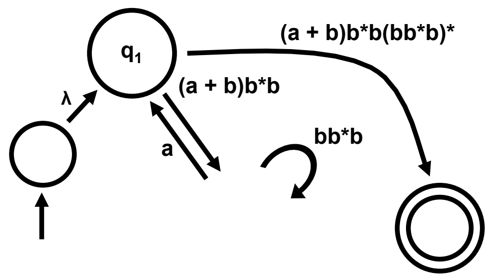
Example 4

DFA→RE

Convert the following NFA into a regular expression?



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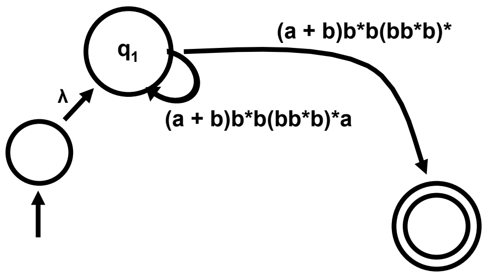
DFA→RE

Example 4

Convert the following NFA into a regular expression?



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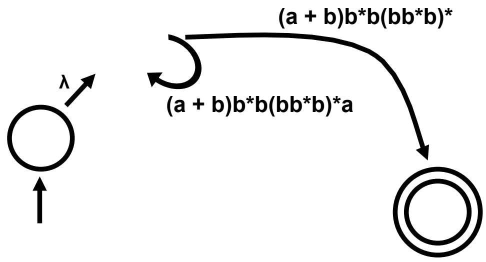
DFA→RE

Example 4

Convert the following NFA into a regular expression?



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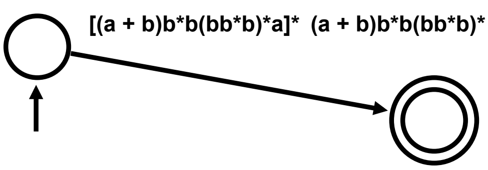
DFA→RE

Example 4

Convert the following NFA into a regular expression?



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DFA→RE

Algorithm

Next we study the DFA→RE Algorithm

Algorithm

Add q_{start} and q_{final} to create G

Run CONVERT(G):

If #states = 2

return the expression on the arrow going from q_{start} to q_{final}

If #states > 2

select $q_{rip} \in Q$ different from q_{start} and q_{final}

define $Q' = Q - \{q_{rip}\}$

define R′ as:

$R'(q_i,q_j) = R(q_i,q_{rip})R(q_{rip},q_{rip})^*R(q_{rip},q_j) \cup R(q_i,q_j)$

return CONVERT(G′)

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Algorithm

CONVERT(G) is equivalent to G

Proof by induction on k (number of states in G)

Base Case:

✓ k = 2

Inductive Step:

Assume claim is true for k-1 states

We first note that G and G′ are equivalent

But, by the induction hypothesis, G′ is equivalent to CONVERT(G′)

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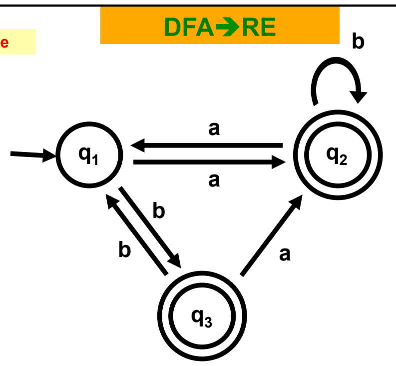
Exercise

DFA→RE



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