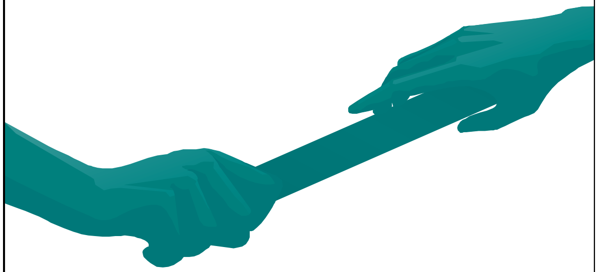
Automata and Languages

Prof. Mohamed Hamada

Software Engineering Lab.
The University of Aizu
Japan

FINITE STATE MACHINES (AUTOMATA)

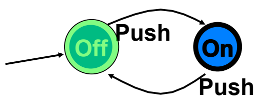


Switch Example 1

Think about the On/Off button

Switch Example 1

The corresponding Automaton



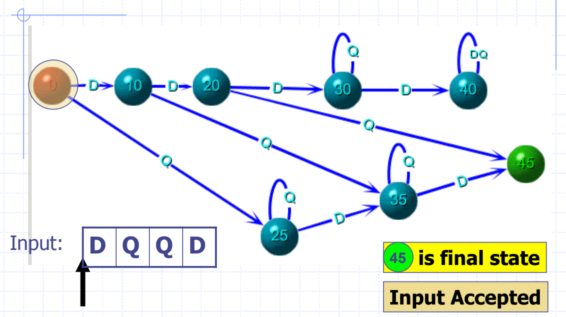
Input:

Push	Push	Push	Push

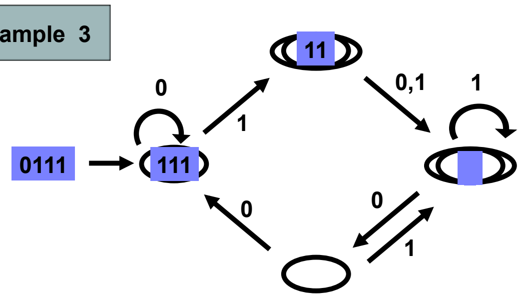
Vending Machine Example 2

◆Vending machine dispenses Cola for $0.45

Vending Machine Example 2

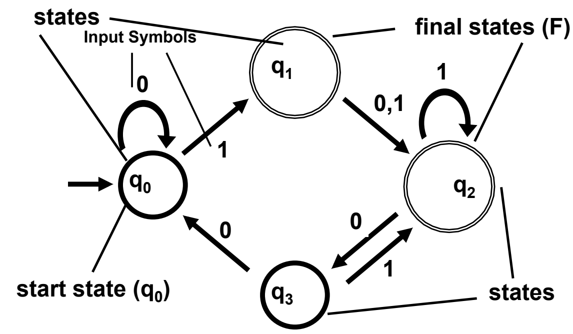


Example 3



The machine accepts a string if the process ends in a *final state*

Example 3



Definitions

An ***alphabet*** **Σ is a finite set of symbols (in Ex3, Σ = {0,1})**

A *string* over Σ is a finite sequence of elements of Σ (e.g. 0111)

For a string s, |s| is the *length* of s

The unique string of length 0 will be denoted by λ and will be called the *empty string*

The ***reversal*** of a string *u* is denoted by u^R.
Example: $(\text{banana})^R$ = ananab

Definitions

The *concatenation* of two strings is the string resulting from putting them together from left to right. Given strings *u* and *v*, denote the concatenation by *u .v*, or just *uv.*

Example:

jap . an = japan, QQ . DD = QQDD

Q1: What's the Java equivalent of concatenation?

The + operator on strings

Q2: Find a formula for |*u .v* |?

|*u .v* | = |*u* |+|*v* |

Definitions

If Σ is an alphabet,
Σ * denotes the set of all strings over Σ.

A *language* over Σ is a subset of Σ *

i.e. a set of strings *each* consisting of sequences of symbols in Σ.

Examples

Example1: in our vending machine we have

Σ = { D, Q }
Σ* = {λ,
D, Q,
DD, DQ, QD, QQ,
DDD, DDQ, DQD, DQQ, QDD, QDQ, QQD, QQQ,
DDDD, DDDQ, … }
L = { *u*∈Σ* | *u* successfully vends }

Example2: in our switch example we have

Σ = { Push}
Σ* = {λ,
Push,
Push Push,
Push Push Push,
Push Push Push Push, … }
L = { Push^n | n is odd }

Definitions

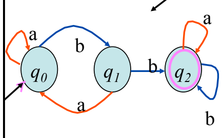
A finite automaton is a 5-tuple $M = (Q, \Sigma, \delta, q_0, F)$

- Q is the set of states
- Σ is the alphabet
- δ is the transition function
- $q_0 \in Q$ is the start state
- $F \subseteq Q$ is the set of final states

L(M) = the language of machine M
= set of all strings machine M accepts

Definitions

State Diagram and Table



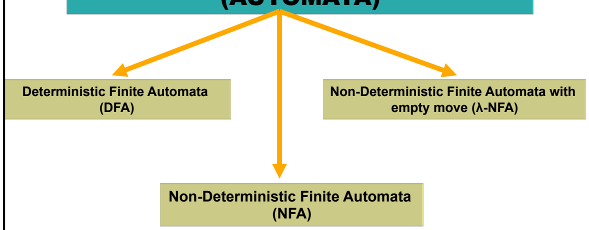
$Q = \{q_0, q_1, q_2\}$

$\Sigma = \{a, b\}$

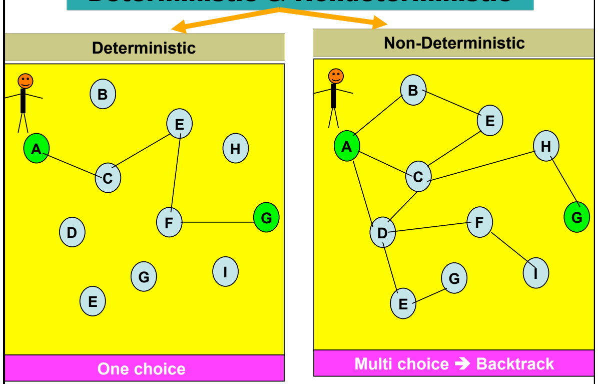
$F = \{q_2\}$

δ	a	b
q_0	q_0	q_1
q_1	q_0	q_2
q_2	q_2	q_2

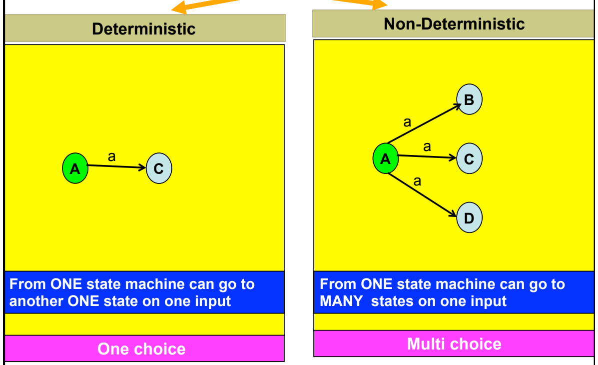
FINITE STATE MACHINES (AUTOMATA)

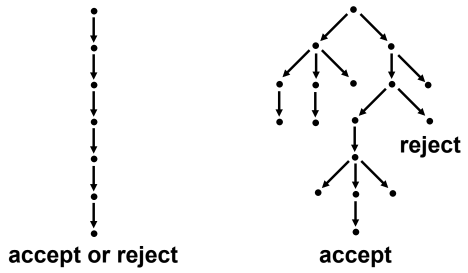


Deterministic & Nondeterministic

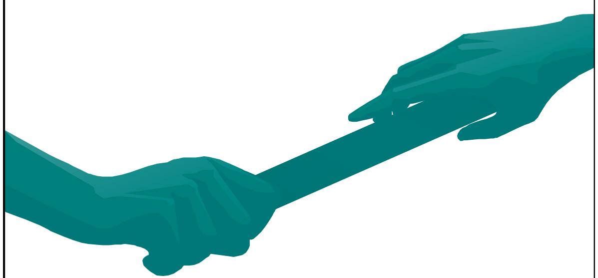


Deterministic & Nondeterministic





DETERMINISTIC FINITE AUTOMATA (DFA)



Definitions

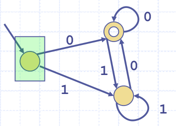
A DFA is a 5-tuple $M = (Q, \Sigma, \delta, q_0, F)$

- Q is the set of states
- Σ is the alphabet
- $\delta : Q \times \Sigma \to Q$ is the transition function
- $q_0 \in Q$ is the start state
- $F \subseteq Q$ is the set of accept states

$L(M)$ = the language of machine M
= set of all strings machine M accepts

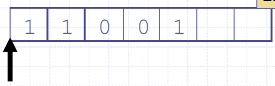
Deterministic Finite Automata (DFA)

Example 1



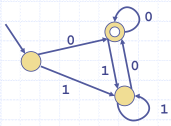
is not final state

Input Rejected



Deterministic Finite Automata (DFA)

Example 1

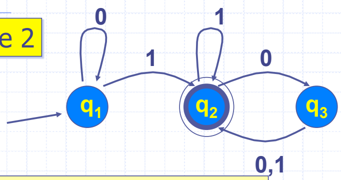


Q: What kinds of bit-strings are accepted?

A: Bit-strings that represent binary even numbers.

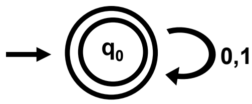
Deterministic Finite Automata (DFA)

Example 2



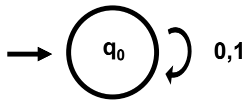
010	reject	
11	accept	
010100100100100		accept
010000010010		reject
λ	reject	

Exercise



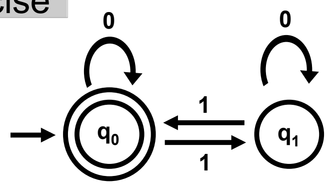
L(M) = {0,1}*

Exercise



L(M) = ∅

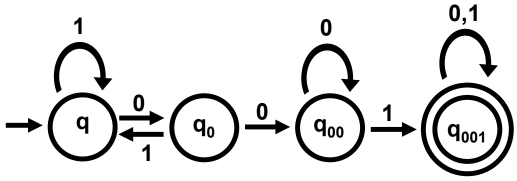
Exercise



L(M) = { w | w has an even number of 1s}

Exercise

Build an automaton that accepts all and only those strings that contain 001



Exercise

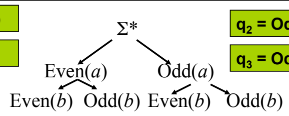
Strings over {*a*,*b*} containing even number of *a*'s and odd number of *b*'s.

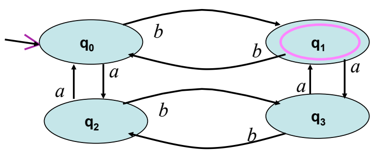
q_0 = Even(*a*).Even(*b*)

q_1 = Even(*a*).Odd(*b*)

q_2 = Odd(*a*).Even(*b*)

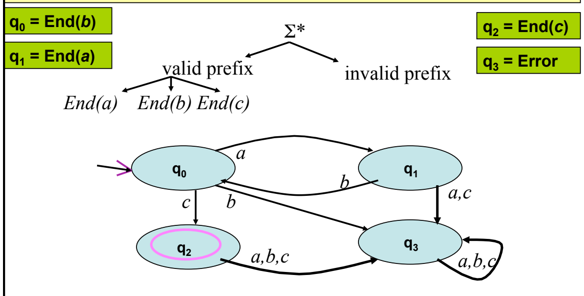
q_3 = Odd(*a*).Odd(*b*)





Exercise

Strings over {*a*,*b*,*c*} that has the form (ab)*c



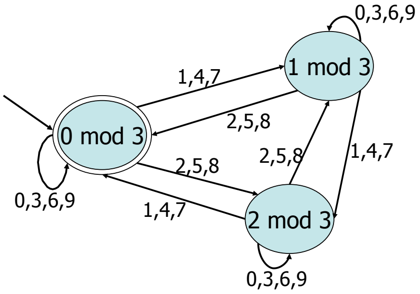
Exercise

Design with a friend a machine that tells us when a *base*-10 number is divisible by 3.

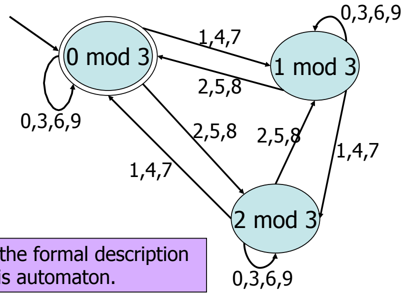
What should your alphabet be?

How can you tell when a number is divisible by 3?

Answer



Exercise



Find the formal description of this automaton.

Answer

Q = { 0 mod 3, 1 mod 3, 2 mod 3 } (rename: {q_0, q_1, q_2})
Σ = { 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 }
q_0 = 0 mod 3
F = { 0 mod 3 }

$\delta : Q \times \Sigma \rightarrow Q$

$\delta(q_0, 2) = q_2,\ \delta(q_0, 9) = q_0, \delta(q_1, 2) = q_0,$

$\delta(q_1, 7) = q_2,\ \delta(q_2, 3) = q_2, \delta(q_2, 5) = q_1.$

Question : $\delta(q_i, j) = ?$

$$\delta(q_i, j) = q_{(i+j) \bmod 3}$$