Answer the following questions.

1. For the set S={1, 4, 7, 8} and the relations:

R1 = {(1,7), (1,8), (7,4), (4,7)}
R2 = {(1,7), (7,1), (7,4), (4,7), (4,4)}
R3 = {(1,1), (1,7), (4,4), (7,1), (7,4), (4,7), (7,7), (8,8)}
R4 = {(1,1), (4,4), (7,7), (8,8)}

a. Complete the following table (by writing yes or no):

	Reflexive	Symmetric	Transitive	Equivalence
R1				
R2				
R3				
R4				

b. Find the following:

i. the reflexive closure for R1 and R2
ii. the transitive closure for R2 and R4
iii. the symmetric closure for R2 and R3
iv. the {reflexive, symmetric, transitive}-closure for R1, and R3.

2. Consider the set S={a, b, c, d, e, f} and the relations:

f1 = {(a,a), (a,b), (c,d), (e,f)}
f2 = {(a,b), (b,c), (c, d), (e,d)}
f3 = {(a,a),(b,b), (c,c), (d,d), (e,e), (f,f)}
f4 = {(a,f),(b,b), (c,d), (e,f)}

Complete the following table (by writing yes or no):

	Function	1-to-1	Onto	1-to-1 correspondence
f1				
f2				
f3				
f4				

3. Prove by induction on *n* that: $\sum_{i=0}^{n} i^3 = (\sum_{i=0}^{n} i)^2$

4. Prove that for a finite set A, $|2^A| = 2^{|A|}$

5. Show that if S_1 and S_2 are finite sets with $|S_1|=n$ and $|S_2|=m$, then $|S_1 \cup S_2| \leq n+m$